

# Variants of Turing Machines

## Variants of TMs

- TMs that can stay put.  
 $\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R, S\}$
- TMs with two-way infinite tape
- Multi-tape TMs
- Nondeterministic TMs
- Enumerators
- All these models are equivalent in power.

## Multi-tape TMs

- Much easier to program with.
- Several tapes, each has its own head for reading and writing.

$$\delta : Q \times \Gamma^k \longrightarrow Q \times \Gamma^k \times \{L, R\}^k$$

$$\delta(q_i, a_1, \dots, a_k) = (q_j, b_1, \dots, b_k, L, R, \dots, L)$$

- Theorem: Every multitape TM has an equivalent single tape TM.  
Proof. Simulate a multitape TM with a single tape TM:  
use one tape to represent all tapes.

## Nondeterministic TMs

- Nondeterministic computation — several possibilities to proceed

$$\delta : Q \times \Gamma \longrightarrow 2^{Q \times \Gamma \times \{L,R\}}$$

$$\delta(q_i, a_1) = \{(q_j, b_1), (q_k, c_1), \dots\}$$

- Theorem: Every nondeterministic TM has an equivalent deterministic TM.

Proof. Simulate a nondeterministic TM with a deterministic TM:  
try all possible derivations.

## Enumerators

- A TM with an attached printer.
  - Start with a blank input tape
  - The language enumerated is all the strings it prints out.