# Significance-Linked Connected Component Analysis for Low Bit Rate Image Coding

Bing-Bing Chai Jozsef Vass Xinhua Zhuang
Department of Computer Engineering and Computer Science
University of Missouri-Columbia
Columbia, MO 65211

**Abstract**

*Recent success in wavelet image coding is mainly attributed to recognition of the importance of data organization and representation. There have been several very competitive wavelet coders developed, namely, Shapiro's embedded zerotree wavelets (EZW), Servetto et al.'s morphological representation of wavelet data (MRWD), and Said and Pearlman's set partitioning in hierarchical trees (SPIHT). In this paper, we develop a novel wavelet image coder called significance-linked connected component analysis (SLCCA) of wavelet coefficients that extends MRWD by exploiting both within-subband clustering of significant coefficients and cross-subband dependency in significant fields. Extensive computer experiments on both natural and texture images show convincingly that the proposed SLCCA outperforms EZW, MRWD, and SPIHT. For example, for the "Barbara" image, at 0.5 bpp SLCCA outperforms EZW and SPIHT by 1.75 dB and 0.89 dB in PSNR, respectively. This outstanding performance is achieved without using any optimal bit allocation procedure, thus both the encoding and decoding procedures are fast.*

## 1 Introduction

Conventional wavelet or subband image coders [1, 2] mainly exploit the energy compaction property of subband decomposition by using optimal bit allocation strategies. The drawback is apparent in that all zero-valued wavelet coefficients, which convey little information, must be represented and encoded, biting away a significant portion of the bit budget. Although this type of wavelet coders provide superior visual quality by eliminating the blocking effect in comparison to block-based image coders such as JPEG, their objective performance measured by PSNR increases only moderately.

Two important issues in wavelet coding are: i) What is the statistical distribution of a wavelet-transformed image within or across subbands, and ii) how to take advantage of the statistical properties of a wavelet-transformed image? Empirically, it has been observed that a wavelet-transformed image as the following statistical properties:

- spatial-frequency localization,
- energy compaction,
- within-subband clustering of significant coefficients,
- cross-subband similarity,
- decaying of magnitude of wavelet coefficients cross subbands.

In recent years, we have seen an impressive advance in wavelet or subband image coding. The success was mainly attributed to the innovative strategies for data organization and representation of wavelet coefficients which exploit not only the energy compaction but also other important statistical properties of wavelet transform. There were three such wavelet image coders published, namely, Shapiro's embedded zerotree wavelet coder (EZW) [3], Servetto *et al.*'s morphological representation of wavelet data (MRWD) [4], and Said and Pearlman's set partitioning in hierarchical trees (SPIHT) [5]. They are all based on empirical observations of the statistical distributions of wavelet-transformed images in one aspect or another. Both EZW and SPIHT exploit cross-subband dependency of insignificant coefficients while MRWD does within subband clustering of significant coefficients. The PSNR of reconstructed images using such data organization strategies was raised by 1–3 dB over block-based transform coders.

In this paper, we propose a novel and more efficient data representation strategy for wavelet image coding termed *significance-linked connected component analysis* (SLCCA). SLCCA strengthens MRWD by exploiting both *within-subband clustering* of significant coefficients and *cross-subband dependency* among significant fields. The cross-subband dependency is effectively exploited by using the so-called significance-link between a parent cluster and a child cluster.

The rest of the paper is organized as follows. Our wavelet image coding algorithm, SLCCA, is presented in the next section. In Section 3, the performance of SLCCA is evaluated against three other wavelet coders, i.e., EZW, MRWD, and SPIHT. The last section concludes the paper.

## 2 Significance-Linked Connected Component Analysis

The key components of SLCCA include multiresolution discrete wavelet image decomposition, connected component analysis of significant fields within subbands, and significance-link registration across subbands, as well as bit plane encoding of magnitudes of significant coefficients by adaptive arithmetic coding.

### 2.1 Formation of Connected Components within Subbands

Since a rather large portion of wavelet field appears insignificant and significant coefficients within subbands tend to be more clustered (Fig. 1), organizing and representing each subband as irregular shaped clusters of significant coefficients provides an efficient way for encoding. Clusters are progressively constructed by using conditioned dilation, resulting in an effective segmentation of the within-subband significant field.

Suppose $A$ is a binary image, $B$ a binary structuring element, and $M \subset A$ a marker. Then, the *conditioned dilation* is defined as

$$D^1(M, A) = (M \oplus B) \cap A,$$

where $\oplus$ denotes the morphological dilation and $\cap$ the intersection. Let

$$D^n(M, A) = D^1(D^{n-1}(M, A), A).$$

Then $D^{\infty}(M, A)$ defines a cluster in $A$. For a digital image, the cluster is formed in finite number of iterations when $D^n(M, A) = D^{n-1}(M, A)$.

In the case of clustering in wavelet field, the binary image $A$ represents the significance map, i.e.,

$$A[x, y] = \begin{cases} 1, & \text{if the wavelet coefficient at location} \\ & \text{[x,y] is significant,} \\ 0, & \text{otherwise.} \end{cases}$$

The marker $M \subset A$ represents the seed of a cluster.

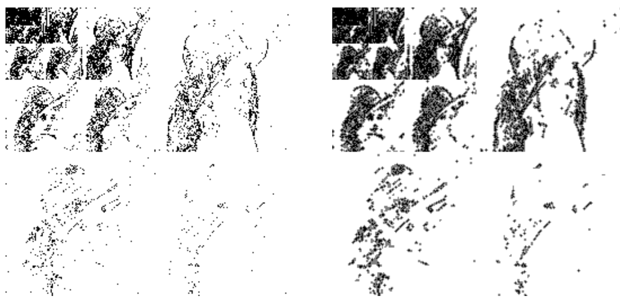
Traditionally, a connected component is defined based on one of the three types of connectivity: 4-connected, 8-connected, and 6-connected, each requiring geometric adjacency of two neighboring pixels. Since the significant coefficients in wavelet field are only loosely clustered, the conventional definition of connected component will produce too many components, affecting the coding efficiency. Thus we may use symmetric structuring elements with a size larger than $3 \times 3$ square. But we still call the segments generated by conditioned dilation *connected components* even if they are not geometrically connected.

To effectively delineate a cluster of significant coefficients, all zero coefficients within the neighborhood $B$ of each significant coefficient in the cluster need to be coded as the boundary of the cluster. By increasing the size of the structuring element, the number of connected components decreases. On the other hand, a larger structuring element results in more boundary zero coefficients. The optimal choice of the size of the structuring element is determined by the cost of encoding boundary zeros versus that of encoding the positional information of connected components. Since the *significance-link* largely reduces the positioning cost, relatively smaller structuring elements can be selected for connected component analysis.

The connected component analysis is illustrated in Fig. 1. The significance map obtained by quantizing all wavelet coefficients with a uniform scalar quantizer of step size $q = 11$ is shown in Fig. 1a. The 22748 significant wavelet coefficients form 1654 clusters using a $5 \times 5$ structuring element. After removing connected components having only one significant coefficient, the number of clusters is reduced to 689. The final encoded significance map is shown in Fig. 1b. It is clear that only a small fraction of zero coefficients are encoded.

### 2.2 Significance-Link in Wavelet Pyramid

The cross-subband similarity among *insignificant coefficients* in wavelet pyramid has been exploited in EZW and SPIHT that greatly improves the coding efficiency. On the other hand, it is found that the spatial similarity in wavelet pyramid is not strictly satisfied, i.e., an insignificant parent does not warrant all four children insignificant. The "isolated zero" symbol used in EZW indicates the failure of such a dependency. The similarity described by zerotree in EZW and the similarity described by insignificant all second generation descendents in SPIHT are more of a reality when a large threshold is used. As was stated in [3], when the threshold decreases (for embedding) to a



(a) (b)

Figure 1: Significance map for six-scale wavelet decomposition, $q = 11$. (a) Significance map after quantization: White pixels denote insignificant coefficients and black pixels significant coefficients. (b) The encoded significance map. White pixels denote coefficients that are not encoded. Black and gray pixels denote encoded significant and insignificant wavelet coefficients, respectively.

certain point, the tree structure or set-partitioned-tree structure are no longer efficient.

In the proposed algorithm, as opposed to EZW and SPIHT, we attempt to exploit the spatial similarity among *significant coefficients*. However, we do not seek a very strong parent-child dependency for each and every significant coefficient. Instead, we try to predict the existence of clusters at finer scales. Statistically, the magnitudes of wavelet coefficients decay from a *parent* to its *children*. It implies that in a cluster formed within a fine subband, there likely exists a significant child whose parent at the coarser subband is also significant. In other words, a significant child can likely be traced back to its parent through this *significance linkage*. It is crucial to note that this significance linkage relies on a much looser spatial similarity.

Formally, two connected components or clusters are called *significance-linked* if the significant parent belongs to one component, and at least one of its children is significant and lies in another component. If the positional information of the significant parent in the first component is available, the positional information for the second component can be inferred through marking the parent as having a significance-link. Since there are generally many significant coefficients in connected components, the likelihood of finding significance-link between two components is fairly high.

### 2.3 Bit-Plane Organizing and Adaptive Arithmetic Coding

As in most image compression algorithms, the last step of SLCCA involves adaptive arithmetic coding [6]. Entropy coding techniques attempt to exploit the source statistics in order to generate an average codeword length closer to the source entropy. In order to exploit the full strength of an adaptive arithmetic coder, it is preferable to organize outcomes of a non-stationary source such as natural images into such a stream that each local probability distribution is in favor of one source symbol. This is the basic idea behind the well known lossless bit-plane coding. Since more significant bit-planes generally contain large uniform areas, the entropy coding techniques can be more efficient.

This idea is employed by SLCCA to encode the magnitude of significant coefficients in each subband. The magnitude of each significant coefficient is converted into a binary representation with a fixed length determined by the maximum magnitude in the subband. Generally, most magnitudes in the subband are smaller than their maximum, implying that more significant bit-planes would contain a lot more 0's than 1's. Accordingly, the adaptive arithmetic coder would generate more accurate local probability distributions in which the conditional probabilities for "0" symbols are closer to one for more significant bit-planes. The context used to define conditional probability models at each significant coefficient is related to the significance status of its eight neighbors and parent.

## 3 Performance Evaluation

The SLCCA is evaluated on several natural $512 \times 512$ grayscale images. The performance is compared with the best wavelet coders EZW, MRWD, and SPIHT. Each image is decomposed into a six-scale subband pyramid. There is no optimal bit allocation carried out in SLCCA. Instead, all wavelet coefficients are quantized with the same uniform scalar quantizer. All the reported bit rates are computed from the actual file sizes.

Table 1 shows the comparison among four wavelet coders at different bit rates. Other results are available at our web site http://www.cecs.missouri.edu/~dcmms. For "Lena," SLCCA consistently outperforms EZW, MRWD, and SPIHT by 1.07 dB, 0.53 dB and 0.18 dB on average, respectively. For "Barbara," on average, SLCCA is superior to EZW by 1.67 dB, and to SPIHT by 0.62 dB.

It appears that SLCCA performs significantly bet-

| Image | Rate [bpp] Algorithm | 0.125 | 0.25 | 0.50 | 1.00 |
|---|---|---|---|---|---|
| Lena | EZW | 30.23 | 33.17 | 36.28 | 39.55 |
| | MRWD | - | - | 36.60 | 40.17 |
| | SPIHT | 31.09 | 34.11 | 37.21 | 40.41 |
| | SLCCA | 31.38 | 34.33 | 37.38 | 40.44 |
| Bar-bara | EZW | 24.03 | 26.77 | 30.53 | 35.14 |
| | SPIHT | 24.86 | 27.58 | 31.39 | 36.41 |
| | SLCCA | 25.45 | 28.43 | 32.28 | 37.15 |

Table 1: Performance comparison (PSNR [dB]) of different wavelet coding algorithms.

ter than SPIHT for images which are rich in texture such as "Barbara" (see Fig. 2) For images which are relatively smooth ("Lena"), the performance between SLCCA and SPIHT gets closer. To further verify the above observation, we compare the performance of SLCCA and SPIHT on eight typical $256 \times 256$ grayscale texture images. The obtained results indicate that SLCCA constantly outperforms SPIHT by 0.32–0.70 dB. An explanation is that when textured images are encoded, wavelet transform is unlikely to yield many large zero regions for lack of homogeneous regions. Thus, the advantage of using an insignificant tree as in EZW, or an insignificant part-of-tree structure as in SPIHT is weakened. On the other hand, SLCCA uses significance-based clustering and significance-based between-cluster linkage, which are not affected by the existence of textures.

(a)

(b)

Figure 2: (a) Original $512 \times 512$ "Barbara" image. Reconstructed image at 0.5 bpp, PSNR=32.28 dB.

## 4 Conclusions

A new image coding algorithm termed significance-linked connected component analysis is presented in this paper. The algorithm takes advantage of two properties of the wavelet decomposition: the within-subband clustering of significant coefficients and the cross-subband dependency in significant fields. Extensive computer experiments justify that SLCCA surpasses the state-of-the-art image coding algorithms reported in the literature. As no optimization is involved, both the encoding and decoding procedures are remarkably fast.

## References


[1] M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies, "Image coding using wavelet transform," *IEEE Transactions on Image Processing*, vol. 1, no. 2, pp. 205–220, Apr. 1992.

[2] N. Farvardin and N. Tanabe, "Subband image coding using entropy-coded quantization," in *Proceedings of SPIE Image Processing Algorithms and Techniques*, 1990, vol. 1244, pp. 240–254.

[3] J.M. Shapiro, "Embedded image coding using zerotrees of wavelet coefficients," *IEEE Transactions on Signal Processing*, vol. 41, no. 12, pp. 3445–3462, Dec. 1993.

[4] S. Servetto, K. Ramchandran, and M.T. Orchard, "Wavelet based image coding via morphological prediction of significance," in *Proceedings of IEEE International Conference on Image Processing*, Oct. 1995, pp. 530–533.

[5] A. Said and W.A. Pearlman, "A new, fast, and efficient image codec based on set partitioning in hierarchical trees," *IEEE Transactions on Circuits and Systems for Video Technology*, vol. 6, no. 3, pp. 243–250, June 1996.

[6] I.H. Witten, M. Neal, and J.G. Cleary, "Arithmetic coding for data compression," *Communications of ACM*, vol. 30, no. 6, pp. 520–540, June 1987.